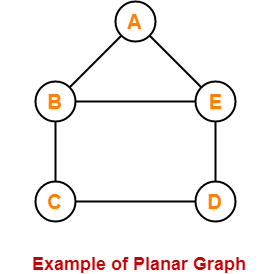
**Planar Graph-**

A planar graph may be defined as-

|  |
| --- |
| In graph theory,  Planar graph is a graph that can be drawn in a plane such that none of its edges cross each other. |

**Planar Graph Example-**

The following graph is an example of a planar graph-



Here,

* In this graph, no two edges cross each other.
* Therefore, it is a planar graph.

**Regions of Plane-**

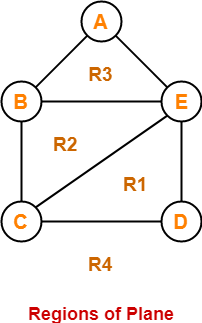
The planar representation of the graph splits the plane into connected areas called as **Regions of the plane**.

Each region has some degree associated with it given as-

* Degree of Interior region = Number of edges enclosing that region
* Degree of Exterior region = Number of edges exposed to that region

**Example-**

Consider the following planar graph-



Here, this planar graph splits the plane into 4 regions- R1, R2, R3 and R4 where-

* Degree (R1) = 3
* Degree (R2) = 3
* Degree (R3) = 3
* Degree (R4) = 5

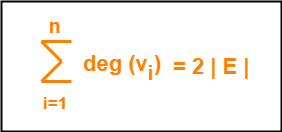
**Planar Graph Chromatic Number-**

* Chromatic Number of any planar graph is always less than or equal to 4.
* Thus, any planar graph always requires maximum 4 colors for coloring its vertices.

**Planar Graph Properties-**

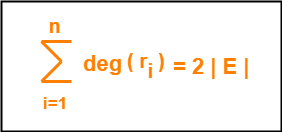
**Property-01:**

In any planar graph, Sum of degrees of all the vertices = 2 x Total number of edges in the graph



**Property-02:**

In any planar graph, Sum of degrees of all the regions = 2 x Total number of edges in the graph



|  |  |  |  |
| --- | --- | --- | --- |
| **Special Cases**    **Case-01:**    In any planar graph, if degree of each region is K, then-     |  | | --- | | **K x |R| = 2 x |E|** |     **Case-02:**    In any planar graph, if degree of each region is at least K (>=K), then-     |  | | --- | | **K x |R| <= 2 x |E|** |     **Case-03:**    In any planar graph, if degree of each region is at most K (<=K), then-     |  | | --- | | **K x |R| >= 2 x |E|** | |

**Property-03:**

If G is a connected planar simple graph with ‘e’ edges, ‘v’ vertices and ‘r’ number of regions in the planar representation of G, then-

|  |
| --- |
| **r = e – v + 2** |

This is known as **Euler’s Formula**.

It remains same in all the planar representations of the graph.

**Property-04:**

If G is a planar graph with k components, then-

|  |
| --- |
| **r = e – v + (k + 1)** |

**PRACTICE PROBLEMS BASED ON PLANAR GRAPH IN GRAPH THEORY-**

**Problem-01:**

Let G be a connected planar simple graph with 25 vertices and 60 edges. Find the number of regions in G.

**Solution-**

Given-

* Number of vertices (v) = 25
* Number of edges (e) = 60

By Euler’s formula, we know r = e – v + 2.

Substituting the values, we get-

Number of regions (r)

= 60 – 25 + 2

= 37

Thus, Total number of regions in G = 37.

**Problem-02:**

Let G be a planar graph with 10 vertices, 3 components and 9 edges. Find the number of regions in G.

**Solution-**

Given-

* Number of vertices (v) = 10
* Number of edges (e) = 9
* Number of components (k) = 3

By Euler’s formula, we know r = e – v + (k+1).

Substituting the values, we get-

Number of regions (r)

= 9 – 10 + (3+1)

= -1 + 4

= 3

Thus, Total number of regions in G = 3.

**Problem-03:**

Let G be a connected planar simple graph with 20 vertices and degree of each vertex is 3. Find the number of regions in G.

**Solution-**

Given-

* Number of vertices (v) = 20
* Degree of each vertex (d) = 3

**Calculating Total Number Of Edges (e)-**

By sum of degrees of vertices theorem, we have-

Sum of degrees of all the vertices = 2 x Total number of edges

Number of vertices x Degree of each vertex = 2 x Total number of edges

20 x 3 = 2 x e

∴ e = 30

Thus, Total number of edges in G = 30.

**Calculating Total Number Of Regions (r)-**

By Euler’s formula, we know r = e – v + 2.

Substituting the values, we get-

Number of regions (r)

= 30 – 20 + 2

= 12

Thus, Total number of regions in G = 12.

**Problem-04:**

Let G be a connected planar simple graph with 35 regions, degree of each region is 6. Find the number of vertices in G.

**Solution-**

Given-

* Number of regions (n) = 35
* Degree of each region (d) = 6

**Calculating Total Number Of Edges (e)-**

By sum of degrees of regions theorem, we have-

Sum of degrees of all the regions = 2 x Total number of edges

Number of regions x Degree of each region = 2 x Total number of edges

35 x 6 = 2 x e

∴ e = 105

Thus, Total number of edges in G = 105.

**Calculating Total Number Of Vertices (v)-**

By Euler’s formula, we know r = e – v + 2.

Substituting the values, we get-

35 = 105 – v + 2

∴ v = 72

Thus, Total number of vertices in G = 72.

**Problem-05:**

Let G be a connected planar graph with 12 vertices, 30 edges and degree of each region is k. Find the value of k.

**Solution-**

Given-

* Number of vertices (v) = 12
* Number of edges (e) = 30
* Degree of each region (d) = k

**Calculating Total Number Of Regions (r)-**

By Euler’s formula, we know r = e – v + 2.

Substituting the values, we get-

Number of regions (r)

= 30 – 12 + 2

= 20

Thus, Total number of regions in G = 20.

**Calculating Value Of k-**

By sum of degrees of regions theorem, we have-

Sum of degrees of all the regions = 2 x Total number of edges

Number of regions x Degree of each region = 2 x Total number of edges

20 x k = 2 x 30

∴ k = 3

Thus, Degree of each region in G = 3.

**Problem-06:**

What is the maximum number of regions possible in a simple planar graph with 10 edges?

**Solution-**

In a simple planar graph, degree of each region is >= 3.

So, we have 3 x |R| <= 2 x |E|.

Substituting the value |E| = 10, we get-

3 x |R| <= 2 x 10

|R| <= 6.67

|R| <= 6

Thus, Maximum number of regions in G = 6.

**Problem-07:**

What is the minimum number of edges necessary in a simple planar graph with 15 regions?

**Solution-**

In a simple planar graph, degree of each region is >= 3.

So, we have 3 x |R| <= 2 x |E|.

Substituting the value |R| = 15, we get-

3 x 15 <= 2 x |E|

|E| >= 22.5

|E| >= 23

Thus, Minimum number of edges required in G = 23.